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Ride Quality of Civil Aircraft from a Probabilistic Viewpoint

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A novel approach is proposed for the analysis and design of the ride quality augmentation system for a civil aircraft. In contrast to the standard problem formulation, where a mean acceleration level is minimized, the ride quality level is quantified in a probabilistic framework. A method is defined to limit the probability of exceeding a given vertical acceleration level in one or more points of the aircraft. The novelty of the proposed approach lies mainly in the definition of the control design in a probabilistic framework, which gives deep physical insight to the problem and more realistic results. An example is discussed to illustrate the proposed approach where the controller is obtained through an eigenstructure assignment procedure; the robustness of the controlled structure is guaranteed through a linear matrix inequality formulation.

Introduction

FLIGHT in atmospheric turbulence places demands from both structural and flight dynamics points of view. These two different areas are joined by the common need to reduce the level of acceleration experienced by the aircraft at proper points of the vehicle. In this paper we are chiefly interested in the second aspect of the problem, which is usually referred to as ride quality optimization. This problem is strictly connected with the comfort of pilot and passengers and arises basically due to the scarce human capability to bear even a low level of acceleration.[1,2]

Typically, the first step toward the definition of a suitable ride quality control system is to evaluate the root mean square value of acceleration at a number of locations on the aircraft. An early approach in this direction has been pursued by Lapins[3] and Jacobson and Lapins.[4] An analytical (and efficient) method to obtain the root means square value of acceleration has been developed by Swaim et al.[5] More recently, using the same line of reasoning, Mengali[6] has proposed a scalar performance index capable of quantifying the effectiveness of the ride quality control system.

The main drawback of the preceding methods is that the quantification of a "mean" acceleration level is able to define the ride quality degree only to a first-order approximation. Indeed, because human discomfort with acceleration is connected to an instantaneous acceleration level (the higher it is, the more intolerable), there is the possibility that acceleration time histories with the same mean value behave differently from a ride quality point of view.

For these reasons, we address the ride quality control problem using a different, new formulation. In the present study the criterion used to quantify the ride comfort is the minimization of the probability of exceeding a certain value of the vertical acceleration variation due to atmospheric turbulence. The substantial improvement with respect to the standard approach should be evident. Indeed, in the context of this new formulation, there is the possibility to obtain not only mean information, but also to quantify the probability that a certain (fixed) acceleration level may be exceeded.

The idea of using probabilistic concepts in control design is not new, but its importance in real-world application has been long underestimated. In the field of structural control, one of the contributions in this area is, for example, the development of a probability of stability analysis using eigenvalue sensitivity.[7,8] Spencer et al.[9] used first- and second-order reliability methods to evaluate the controller robustness. In all of the preceding cases, the probabilistic environment has been mainly used to take into account possible uncertainties in the model of the system to be controlled and to measure the controller robustness. In the area of aerospace guidance and control, much of the literature regarding the discussed problems is due to Stengel,[10] Stengel and Ray,[11] Ray and Stengel,[12,13] and Marrison and Stengel.[14] Starting from a study where the effect of parameter variations on the probability of instability of aircraft has been investigated by means of Monte Carlo computation,[10] the concept of stochastic robustness has been properly formulated[11] and applied to a full state feedback aircraft control system.[12] Concepts behind stochastic robustness have been extended to introduce the stochastic performance robustness idea that has been successfully applied to the design of a longitudinal controller for a jet transport[13] and, recently, to a hypersonic aircraft.[14]

In this paper, however, we used a sharply different approach. The control aim is posed in probabilistic terms in the sense that it is designed to minimize (maximize) the probability that, for the controlled structure, a given event takes place. This probability is in turn evaluated by means of an approach first introduced by Dirlik[15] and then tested for accuracy by Bouyssy et al.[16] A set of acceptable controllers is obtained by formulating the problem as an optimization problem with constraint, the solution of which is easily accomplished by means of standard numerical software. Finally, the robustness of the obtained controllers is evaluated by means of a linear matrix inequality formulation.

The paper is organized as follows: First the model of the aircraft dynamics is given, then the probability distribution function of the variations of the vertical acceleration experienced at a fixed point of the aircraft is described. Next, two different formulations of the problem at hand are addressed, and the problem of controller robustness is considered. A detailed case study then follows.

System Model

The aircraft dynamics are described by the following linear, time-invariant system:

$$\dot{\boldsymbol{x}} = A_0\boldsymbol{x} + B_{0w}\boldsymbol{w}_g + B_{0u}\boldsymbol{u} \tag{1}$$

$$\boldsymbol{z} = C_{0z}\boldsymbol{x} + D_{zw}\boldsymbol{w}_g + D_{zu}\boldsymbol{u} \tag{2}$$

$$\boldsymbol{y} = C_{0y}\boldsymbol{x} + D_{yw}\boldsymbol{w}_g + D_{yu}\boldsymbol{u} \tag{3}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the vector containing the states of the system, $\boldsymbol{u} \in \mathbb{R}^m$ denotes the control variable, $\boldsymbol{w}_g \in \mathbb{R}^r$ contains the gust velocity components, $\boldsymbol{y} \in \mathbb{R}^{p_y}$ is the vector of measured signals, and $\boldsymbol{z} \in \mathbb{R}^{p_z}$ contains the regulated output signals. The atmospheric turbulence is described by means of a linear system driven by white noise,

$$\dot{\boldsymbol{d}}_g = A_g\boldsymbol{d}_g + B_g\eta \tag{4}$$

$$\boldsymbol{w}_g = C_g\boldsymbol{d}_g \tag{5}$$

and the matrices in Eqs. (4) and (5) are chosen in accordance with a Dryden model. The vector $\boldsymbol{d}_g \in \mathbb{R}^s$ contains the states of

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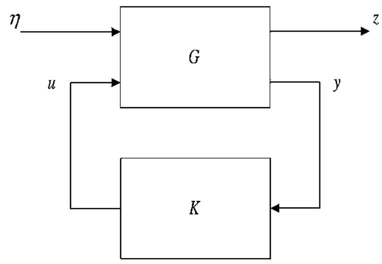

Fig. 1 Control system arrangement.

the atmospheric disturbances and $\eta \in \mathbb{R}^q$ is white noise with constant intensity. Obviously, Eqs. (1–5) may be easily combined by defining an augmented state vector

$$\xi := \begin{Bmatrix} \boldsymbol{x} \\ \boldsymbol{d}_g \end{Bmatrix} \tag{6}$$

with the following result:

$$\dot{\xi} = \begin{bmatrix} A_0 & B_{0w}C_g \\ 0 & A_g \end{bmatrix} \xi + \begin{bmatrix} 0 \\ B_g \end{bmatrix} \eta + \begin{bmatrix} B_{0u} \\ 0 \end{bmatrix} \boldsymbol{u} \tag{7}$$

$$z = [C_{0z} \quad D_{zw}C_g]\xi + D_{zu}\boldsymbol{u} \tag{8}$$

$$\boldsymbol{y} = [C_{0y} \quad D_{yw}C_g]\xi + D_{yu}\boldsymbol{u} \tag{9}$$

Our aim is to find a suitable control law, in the form:

$$\boldsymbol{u} = K\boldsymbol{y} \tag{10}$$

where K is a constant matrix, in such a way that the regulated output signals are "small" according to some prescribed measure. The problem, in this standard form, is summarized in the block scheme of Fig. 1, where G is the generalized plant matrix.

We are interested in taking the regulated output z under control in a probabilistic sense; in other words we want to assess that the probability that z exceeds a certain (prescribed) level is less than a fixed value. This concept is particularly important from a ride quality viewpoint, where the acceleration level must as low as possible. In contrast to a classic approach where only a *mean* acceleration level may eventually be taken under control, our novel formulation allows one to guarantee that a certain acceleration level is not exceeded, at least with a given probability. Of course, the first step toward approaching this problem consists of defining the probability density function of the regulated output. This matter is addressed in the next section.

Probabilistic Formulation

The probabilistic framework for the problem is as follows: The ride quality level is quantified through the probability of not exceeding a given vertical acceleration variation range. It is clear that a probabilistic formulation is a great improvement with respect to a classic approach. Indeed, evaluating a probabilistic performance allows one to obtain more realistic results and, in particular, to overcome the classic concept of producing controllers where controller margins are a worst-case measure of the system performance.

For controller design, the transfer function of the vertical acceleration, for example, $H_{z\eta}(\omega)$, at a fixed point of the aircraft is assumed to be given together with the mission profile, so that the parameters that appear in the Dryden model [the entries of the matrices in Eqs. (4) and (5)] are known. The spectrum of the vertical acceleration $\Phi_{a_z}(\omega)$ can be obtained from

$$\Phi_{a_z}(\omega) = \Phi_\eta(\omega)|H_{z\eta}(\omega)|^2 \tag{11}$$

Note that, because η is white, $\Phi_\eta(\omega)$ is unitary.

At this point one's aim would be to evaluate the probability density function of the variations of the vertical acceleration given its spectrum, so that the probability of occurrence of a given acceleration range can be obtained by integration.

The problem of obtaining a plausible probability density function for the actual variation ranges of a variable whose spectrum is known is of interest in the civil engineering field, where it is often necessary to obtain the probability distribution of the load excursion (given the applied loading spectrum) to evaluate the damage developed in the loaded structure.

Studies have been carried out[16–19] to obtain an analytical counting procedure, starting from the loading spectra, in the case that the process of interest is Gaussian. Various formulations have been tested and proposed; the effectiveness of the different empirical procedures depends, among other things, on the value of the bandwidth parameter α defined as

$$\alpha := m_2 / \sqrt{m_0 m_4} \tag{12}$$

with m_k ($k = 1, \ldots, 4$) being the spectral moment of order k. These moments can be evaluated by means of the following equation:

$$m_k := \int_0^\infty \omega^k \Phi(\omega)\, \mathrm{d}\omega \tag{13}$$

where $\Phi(\omega)$ is the one-sided spectral density function.

Bouyssy et al.[16] have tested many approximating functions for probability density function of the cycle ranges; they found that good results for small α can be obtained using the Dirlik formula,[15] which, in our case, may be written as

$$f(\Delta a_z) = \frac{1}{2}\frac{D_1}{Q}\exp\left(-\frac{\Delta a_z}{2Q}\right) + \frac{\Delta a_z D_2}{4R^2}\exp\left[-\frac{1}{2}\left(\frac{\Delta a_z}{2R}\right)^2\right]$$
$$+ \frac{\Delta a_z D_3}{4}\exp\left[-\frac{1}{2}\left(\frac{\Delta a_z}{2}\right)^2\right] \tag{14}$$

where

$$X_m = (m_1/m_0)\sqrt{m_2/m_4} \tag{15}$$

$$D_1 = 2\frac{X_m - \alpha^2}{1 + \alpha^2} \tag{16}$$

$$R = \frac{\alpha - X_m - D_1^2}{1 - \alpha D_1 + D_1^2} \tag{17}$$

$$Q = D_1/4 \tag{18}$$

$$D_2 = \frac{1 - \alpha - D_1 + D_1^2}{1 - R} \tag{19}$$

$$D_3 = 1 - D_1 - D_2 \tag{20}$$

Because the Dirlik formula is valid for a Gaussian process with zero mean and unit variance, note that Δa_z is an adimensionalized random variable. Actually it is the ratio between the vertical acceleration and the standard deviation of the process at hand.

The preceding relations have been used in the present work to obtain the probability distribution function F for the variations of the vertical acceleration experienced at a fixed point of the aircraft. The probability of not exceeding a given vertical acceleration range, for example, Δa_z^*, is given by the integral of the probability density function, that is,

$$F(\Delta a_z^*) := P(\Delta a_z \le \Delta a_z^*) = \int_0^{\Delta a_z^*} f(\Delta a_z)\, \mathrm{d}\Delta a_z \tag{21}$$

where P is probability. The maximization of Eq. (21) is the goal of the controller design.

Probabilistic Ride Quality Control

To describe how these concepts may be applied to a practical case, we consider the two-degree-of-freedom short-period longitudinal dynamics of a civil aircraft. The regulated output z is the acceleration experienced by the aircraft at the most rearward fuselage station. This point is typically the most uncomfortable from a ride quality point of view.[1,6]

The control system is intended to decrease significantly the probability that the acceleration level, as would be experienced at that point without control, exceeds a prescribed level. However, recall that a number of other constraints must also be taken into account. First, it must be assured that the flight quality requirements be fulfilled; this constrains the poles of the controlled system to lie in a prescribed region of the complex plane. Second, for maneuverability considerations, the control surfaces must be deflected by the control system as little as possible. This requirement may be seen as a further constraint on the pole location. To accommodate the conflicting requirements, we used, as a quite natural choice, an eigenstructure assignment static control law. The control inputs are assumed to be the elevators and the symmetric rotation of ailerons. Also, we assume that the measured outputs are the vertical velocity component w and the pitch rate q. Under these hypotheses, the K matrix in Eq. (10) can be easily evaluated as follows. Note that Eq. (3) becomes

$$y = [I \quad 0]\xi \tag{22}$$

where I is the identity matrix. Also, Eqs. (7) and (8) may be written as

$$\xi = \begin{bmatrix} A_0 + B_{0u}K & B_{0w}C_g \\ 0 & A_g \end{bmatrix}\xi + \begin{bmatrix} 0 \\ B_g \end{bmatrix}\eta \tag{23}$$

$$z = [C_{0z} + D_{zu}K \quad D_{zw}C_g]\xi \tag{24}$$

Note that the aircraft short-period modes are described by the eigenvectors and eigenvalues of the matrix

$$A_c := A_0 + B_{0u}K \tag{25}$$

Suppose the eigenvectors of the closed-loop matrix A_c to be fixed and arranged in columns of the modal matrix M. Because it is always possible to write $A_c = M\Lambda_c M^{-1}$, with Λ_c the (diagonal) matrix with the eigenvalues of the closed-loop system, the closed-loop gain matrix K can be easily derived by the following equation:

$$K = B_{0u}^{-1}(M\Lambda_c M^{-1} - A_0) \tag{26}$$

Obviously, K may be evaluated by means of Eq. (26) when Λ_c has been specified. Note that the particularly simple structure of Eq. (26) is because in our example B_{0u} is square (there are two inputs and two measured outputs). However, in a more complex situation where B_{0u} is not square, it is always possible to use more sophisticated algorithms for eigenstructure assignment (see Andry et al.[20] for an excellent survey).

The problem can be described as follows: Given the acceleration level Δa_z^*, find the controller [in the form of Eq. (10)] that maximizes Eq. (21) such that the eigenstructure of the A_c matrix meets the aircraft flight quality requirements. The steps toward the solution may now be explicitly summarized as follows.

1) Define the acceleration level Δa_z^*.

2) Define the constraints on the eigenvalues and eigenvectors of A_c.

3) With a set of candidate values for the eigenvalues and eigenvectors, evaluate K using Eq. (26).

4) Find the spectrum of the vertical acceleration using Eq. (11). Note that $H_{z\eta}(\omega)$ may be computed using Eqs. (23) and (24).

5) Evaluate the density function with Eq. (14), using Eqs. (12), (13), and (15–20).

6) Find the probability of not exceeding Δa_z^* using Eq. (21).

7) Update the candidate values for eigenvalues and eigenvectors and go to point 3 until $F(\Delta a_z^*)$ has been maximized.

To simplify the discussion and without loss of generality, suppose the values of the eigenvectors are fixed such that they coincide with the corresponding values of the open-loop matrix A_0. Accordingly, only the eigenvalues have to be optimized with the following constraints:

$$\rho_{\min} \le \rho \le \rho_{\max} \qquad \zeta_{\min} \le \zeta \le \zeta_{\max} \tag{27}$$

where ρ and ζ are the modulus and damping coefficient of the closed-loop eigenvalues and the subscripts min and max denote minimum and maximum values attained by the indicated variables. Because the eigenvectors are fixed, it is clear that $F(\Delta a_z^*)$ can be seen as a function of two independent variables only (ρ and ζ) and can be easily maximized with standard numerical software.

To summarize, we have problem 1:

$$\begin{aligned} \text{maximize} \quad & F(\Delta a_z^*) \\ \text{subject to} \quad & \rho_{\min} \le \rho \le \rho_{\max} \\ & \zeta_{\min} \le \zeta \le \zeta_{\max} \end{aligned} \tag{28}$$

Before proceeding further, we propose a second (and probably more useful) problem formulation. To this end, suppose a probability value, for example, P^*, to be fixed, and suppose a related minimum acceptable level of acceleration range Δa_z^*. This is equivalent to stating that a value attained by $P(\Delta a_z \le \Delta a_z^*)$ is acceptable if and only if this value is not less than P^*. With this preliminary statement, it is clear that we may find a (possibly infinite) set of acceptable controllers. In view of the preceding discussion, we decide to define the "best" controller in this set as that corresponding to a situation of minimum actuator activity that in turn corresponds to the minimum Frobenius norm of the controller matrix K. Hence, the problem 2 formulation is as follows:

$$\begin{aligned} \text{minimize} \quad & \|K\| \\ \text{subject to} \quad & F(\Delta a_z^*) \ge P^* \\ & \rho_{\min} \le \rho \le \rho_{\max} \\ & \zeta_{\min} \le \zeta \le \zeta_{\max} \end{aligned} \tag{29}$$

where $\|K\|$ denotes the Frobenius norm of K.

Robustness Considerations

An important step toward the validation of the proposed control law lies in the possibility of assessing how robust the control law is against model uncertainties. We suppose that all uncertainties are due to the aerodynamic derivatives of the aircraft. With reference to the aircraft dynamics summarized in the Appendix, it can be shown that the linearized equations of motion can be written as [see Eq. (A4)]

$$E(\boldsymbol{p})\dot{\boldsymbol{x}} = A_c(\boldsymbol{p})\boldsymbol{x} \tag{30}$$

where $\boldsymbol{p} \in \mathbb{R}^\ell$ is the vector of parameters (which in our case coincide with the aerodynamic derivatives). Note (see the Appendix for details) that $A_c(\boldsymbol{p})$ and $E(\boldsymbol{p})$ may be written as

$$A_c(\boldsymbol{p}) = A_0 + p_1 A_1 + p_2 A_2 + \cdots + p_\ell A_\ell \tag{31}$$

$$E(\boldsymbol{p}) = E_0 + p_1 E_1 + p_2 E_2 + \cdots + p_\ell E_\ell \tag{32}$$

from which it is clear that $A_c(\boldsymbol{p})$ and $E(\boldsymbol{p})$ are affine functions of the vector $\boldsymbol{p}$. Each aerodynamic derivative is assumed to have an uncertain value ranging in a given interval, that is,

$$p_j^{\min} \le p_j \le p_j^{\max} \qquad j = 1, \dots, \ell \tag{33}$$

To assess the stability of the system described by Eq. (30) we use a quadratic Lyapunov function in the form

$$V(\boldsymbol{x}) = \boldsymbol{x}^T T \boldsymbol{x} \tag{34}$$

It is well known that a sufficient condition for the stability of Eq. (30) is the existence of a matrix $T > 0$ such that the inequality

$$\frac{\mathrm{d}V(\boldsymbol{x})}{\mathrm{d}t} < 0 \tag{35}$$

holds along all state trajectories. Now, letting $W := T^{-1}$, it is easily verified that the stability condition is equivalent to the existence of W such that

$$A_c(\boldsymbol{p})WE^T(\boldsymbol{p}) + E(\boldsymbol{p})WA_c^T(\boldsymbol{p}) < 0 \tag{36}$$

Also, by a coordinate transformation, it can be shown[20] that the problem described by Eq. (36) is an eigenvalue problem in the variables γ and $W = W^T$ that for all admissible values of (A_c, E) can be formulated as follows:

$$\begin{aligned}&\text{minimize } \gamma\\ &\text{subject to} \quad A_c(\boldsymbol{p})WE^T(\boldsymbol{p}) + E(\boldsymbol{p})WA_c^T(\boldsymbol{p}) < 0\\ &\qquad\qquad I \le W \le \gamma I\end{aligned} \tag{37}$$

The problem has now been recast into a linear matrix inequality formulation[21] for which reliable software is available.[22] The system is stable as long as γ is negative. Recall, however, that because the preceding conditions are sufficient to test the stability of the system, the latter could be stable also if γ is positive.

Finally, under the further assumption that matrix E does not depend on the parameter vector $\boldsymbol{p}$, it is possible to define the quadratic stability margin. This is the largest nonnegative ε for which the system is quadratically stable when

$$p_j^{\text{nom}} - \varepsilon\delta_j \le p_j \le p_j^{\text{nom}} + \varepsilon\delta_j \qquad j = 1, \ldots, \ell \tag{38}$$

where $p_j^{\text{nom}} = 0.5(p_j^{\max} + p_j^{\min})$ is the nominal value of p_j and $\delta_j = 0.5(p_j^{\max} - p_j^{\min})$ is the radius of the jth interval. The value of ε is computed by solving the following generalized eigenvalue problem[20] in ε and $W = W^T$ for all admissible values of A_c:

$$\begin{aligned}&\text{maximize } \varepsilon\\ &\text{subject to} \quad \varepsilon \ge 0\\ &\qquad A_{c,0}W + WA_{c,0}^T + \varepsilon\left[A_c(\boldsymbol{p})W + WA_c^T(\boldsymbol{p})\right] < 0\end{aligned} \tag{39}$$

where $A_{c,0}$ is the value of A_c attained when $p_j = p_j^{\text{nom}}$.

Case Study

In the example to follow we used the data taken from Mengali,[6] where the dynamics of a DC-8-type aircraft with symmetric aileron deflection is described. This vehicle is trimmed in a cruise condition characterized by an altitude of 33,000 ft and a Mach number equal to 0.84.

In our example we consider a turbulence with rms value of 20 ft/s. According to the U.S. 1967 classification, this corresponds to the limit value between light and moderate turbulence severity.[23] The corresponding standard deviation of vertical acceleration at the reference point is $\sigma_{\text{ol}} = 17.36$ ft/s^2 for the open-loop case. It is well known that a moderate rms value of vertical acceleration may be very annoying for crew and passengers. For example, exposure to a vertical acceleration level of $0.1g$ in a few minutes considerably reduces the comfort of passengers,[2] and the situation is even worse for lateral acceleration intensities. For these reasons we decided to consider the 0.08-g level as the representative point. In view of the preceding discussion, our objective is to increase the probability of not exceeding a given vertical acceleration at the reference point. In the open-loop case the situation is this: the probability of not exceeding the 0.08-g level is equal to $F(0.08g/\sigma_{\text{ol}}) = 0.82$. Our aim is to increase the level to be greater or equal to 0.9. Referring to problem 2 of the "Probabilic Ride Quality Control" section, this is equivalent to solve the following problem:

$$\begin{aligned}&\text{minimize} \quad \|K\|\\ &\text{subject to} \quad F(0.08g/\sigma_{\text{cl}}) \ge 0.9\\ &\qquad 2\ (\text{rad/s}) \le \rho \le 5\ (\text{rad/s})\\ &\qquad 0.3 \le \zeta \le 1\end{aligned} \tag{40}$$

where σ_{cl} is the standard deviation of vertical acceleration for the closed-loop system.

The minimum obtained value of $\|K\|$ is 11.22. Moreover,

$$K = \begin{bmatrix} +0.0025 & -2.9680\\ -0.0127 & +10.8234\end{bmatrix} \tag{41}$$

from which it is easily verified that this is a feasible solution because the constraints assume the values $F(0.08g/\sigma_{\text{cl}}) = 0.9$, $\rho = 1.971$ rad/s, and $\zeta = 0.3$ that agree with Eq. (40). Results in terms of the parameters that define the Dirlik formula have been summarized in Table 1 for the open- and closed-loop cases.

In Figs. 2 and 3 the probability density functions and the probability distribution functions are shown for both the open- and closed-loop systems. Note that for ease of comparison both probabilities have been referred to a dimensional acceleration level that is shown in gravitational acceleration units. From Figs. 2 and 3 it can be easily appreciated how the controller acts on the probability of

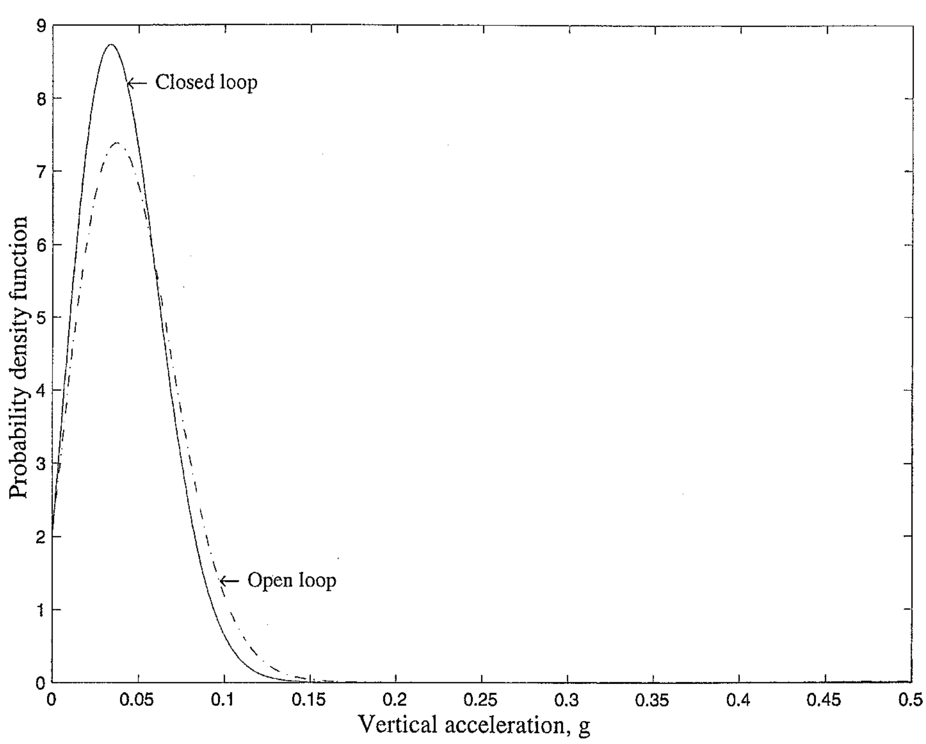

Fig. 2 Probability density functions for the open- and closed-loop systems.

encountering a vertical acceleration excursion range. The result is clear: The probability density function of the closed-loop system is shifted to the left, thus giving higher probability to have lower acceleration levels, which indeed is the actual aim of the controller. Note that the probability distribution for the open-loop system tends to unity more slowly than that of the closed-loop system.

The robustness of the control system against variations of the aerodynamic derivatives has been evaluated. With reference to the earlier described procedure, in Table 2 the nominal values and the interval of variation have been reported. Note that the chosen values correspond to variations of 5% in M_{δ_a} and M_{δ_e} and 10% in all of the other aerodynamic derivatives. It may be verified that the controlled system is stable for the assumed design variations in the aerodynamic derivatives.

Further study has been carried out to compute the maximum range allowable in the nominal values of the aerodynamic derivatives such that the stability of the closed-loop system is still guaranteed. In so doing we made the further assumption that $M_{\dot{w}}$ coincides with its nominal value. Applying the described procedure, it has been found that a further 152% extension of the interval of variations is allowed.

Table 1 Open- and closed-loop values attained by the parameters of the Dirlik formula

Parameter	Open-loop value	Closed-loop value
X_m	0.0648	0.0347
D_1	0.1033	0.0591
D_2	0.8255	0.9031
D_3	0.0711	0.0378
Q	0.0258	0.0148
R	0.0364	0.0322
α	0.1118	0.0704

Table 2 Nominal, minimum, and maximum values attained by the parameter vector

Parameter	Nominal value	Minimum value	Maximum value
Z_w	−0.806	−0.7657	−0.8463
M_w	−0.0111	−0.0105	−0.0117
M_q	−0.924	−0.8778	−0.9702
Z_{δ_e}	−34.6	−32.87	−36.33
Z_{δ_a}	−37.2	−35.34	−39.06
M_{δ_e}	−4.59	−4.4752	−4.7048
M_{δ_a}	−1.219	−1.1885	−1.2495
$M_{\dot{w}}$	−0.00051	$-4.845E-4$	$-5.355E-4$

Conclusions

A control strategy has been proposed to guarantee that a minimum fixed value of the probability of encountering a given vertical acceleration level is met. This result allows one to revise the classical concept of ride quality level, which no longer needs to be based on a definition of a mean level of acceleration experienced by the aircraft in one or more points along the fuselage. The described approach has been verified through a numerical example to possess very interesting practical consequences and may be effectively used in the control system design phase. Important other aspects, such as the evaluation of the controller robustness, have been investigated both theoretically and numerically by means of a linear matrix inequality formulation to take into account the unavoidable uncertainties in the determination of the aerodynamic derivatives. The maximum allowed range of variation of the aerodynamic derivatives has also been evaluated for the sample case where $M_{\dot{w}}$ has been assumed to keep its nominal value.

The proposed approach is valuable because it represents the first step toward properly taking the randomness and the uncertainties of the design variables into account. In a more general probabilistic framework, other uncertainties in the model definition can also be introduced in Eq. (21). This subject is currently being investigated.

Appendix: Equations of motion

The open-loop short-period equations of the vehicle dynamics are given by

$$\begin{bmatrix} 1 & 0 \\ -M_{\dot{w}} & 1 \end{bmatrix} \begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_w & U_0 \\ M_w & M_q \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} & Z_{\delta_a} \\ M_{\delta_e} & M_{\delta_a} \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_a \end{bmatrix} \quad \text{(A1)}$$

where U_0 is the flight velocity. From Eq. (25) one has

$$A_c = \begin{bmatrix} Z_w & U_0 \\ M_w & M_q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} & Z_{\delta_a} \\ M_{\delta_e} & M_{\delta_a} \end{bmatrix} \begin{bmatrix} K_1 & K_2 \\ K_3 & K_4 \end{bmatrix} \quad \text{(A2)}$$

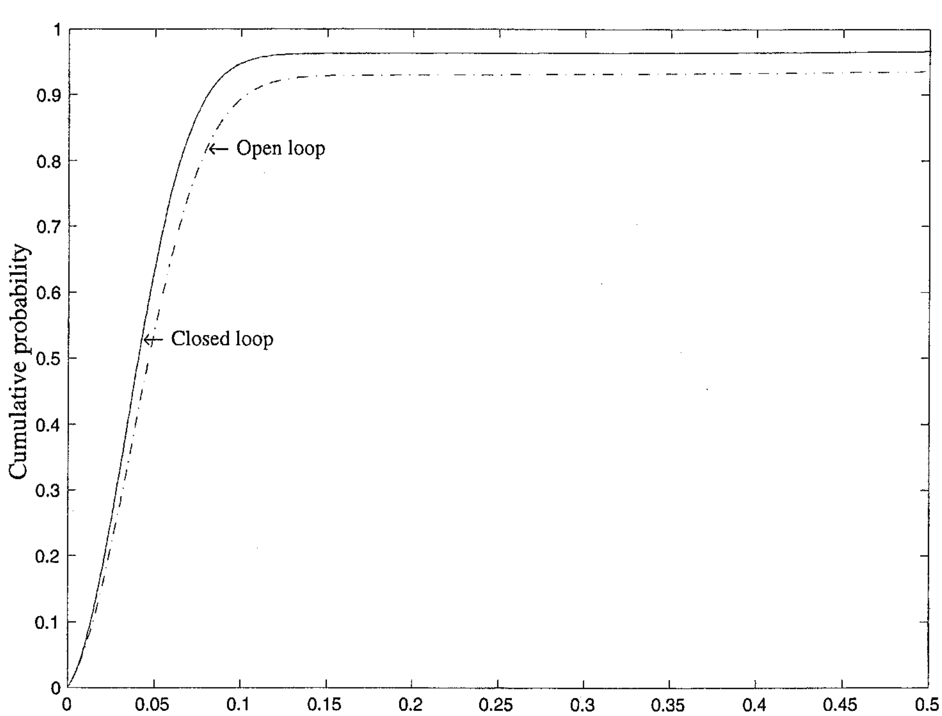

Fig. 3 Probability distribution functions for the open- and closed-loop systems.

where K_j, $j = 1, \ldots, 4$, are the four entries of K, or

$$A_c = \begin{bmatrix} Z_w + Z_{\delta_e} K_1 + Z_{\delta_a} K_3 & U_0 + Z_{\delta_e} K_2 + Z_{\delta_a} K_4 \\ M_w + M_{\delta_e} K_1 + M_{\delta_a} K_3 & M_q + M_{\delta_e} K_3 + M_{\delta_a} K_4 \end{bmatrix} \tag{A3}$$

Clearly, Eq. (A1) may be written in the form

$$E(\boldsymbol{p})\dot{\boldsymbol{x}} = A_c(\boldsymbol{p})\boldsymbol{x} \tag{A4}$$

where $\boldsymbol{p}$ is the vector of parameters. In our case we have

$$\boldsymbol{p} := (Z_w, M_w, M_q, M_{\dot{w}}, Z_{\delta_e}, Z_{\delta_a}, M_{\delta_e}, M_{\delta_a}) \tag{A5}$$

Note that $A_c(\boldsymbol{p})$ and $E(\boldsymbol{p})$ may be easily written as

$$A_c(\boldsymbol{p}) = A_0 + p_1 A_1 + p_2 A_2 + \cdots + p_\ell A_\ell \tag{A6}$$

$$E(\boldsymbol{p}) = E_0 + p_1 E_1 + p_2 E_2 + \cdots + p_\ell E_\ell \tag{A7}$$

from which it is seen that the closed-loop system is affine.

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